Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

Thursday 17 May 2018 – Afternoon

Paper Reference
8FM0/21

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1/1/1/1/1/

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that the equation

$$5\sin x + 12\cos x = 2$$

can be written in the form

$$7t^2 - 5t - 5 = 0$$

(3)

(b) Hence solve, for $-180° < x < 180°$, the equation

$$5\sin x + 12\cos x = 2$$

giving your answers to one decimal place.

(4)

a)

$$5\sin x + 12\cos x = 2$$

$\frac{\sin(\frac{x}{2})}{\cos(\frac{x}{2})} = \tan(\frac{x}{2})$

$2\sin(\frac{x}{2})\cos(\frac{x}{2}) = \sin x$

$= 2t\cos^2(\frac{x}{2})$

$= 2t\sec^2(\frac{x}{2})$

$1 + \tan^2\theta \equiv \sec^2\theta$

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$\Rightarrow 5\left(\frac{2t}{1+t^2}\right) + 12\left(\frac{1-t^2}{1+t^2}\right) = 2$$

$\times(1+t^2)$

$$\Rightarrow 10t + 12 - 12t^2 = 2(1+t^2)$$

$$\Rightarrow 2t^2 + 2 + 12t^2 - 10t - 12 = 0$$

$$\Rightarrow 14t^2 - 10t - 10 = 0$$

$$\div 2: \Rightarrow 7t^2 - 5t - 5 = 0$$

b) solving $7t^2 - 5t - 5 = 0$:

Quadratic Formula : $a = 7, b = -5, c = -5$

Question 1 continued

$\therefore t = \dfrac{5 \pm \sqrt{165}}{14} = \tan\left(\dfrac{x}{2}\right)$

$\dfrac{x}{2} = \tan^{-1}\left(\dfrac{5-\sqrt{165}}{14}\right)$

$\dfrac{x}{2} = -0.5108$

$x = -1.0215\text{rad}$
$= -58.5°$

$\dfrac{x}{2} = \tan^{-1}\left(\dfrac{5+\sqrt{165}}{14}\right)$

$\dfrac{x}{2} = 0.9056$

$x = 1.8111\text{ rad}$
$= 103.8°$

secondary values won't fit in range

$\therefore$ final values are $x = 103.8°$ & $x = -58.5°$

(Total for Question 1 is 7 marks)

2. The temperature, θ°C, of coffee in a cup, t minutes after the cup of coffee is put in a room, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 20)$$

where k is a constant.

The coffee has an initial temperature of 80 °C

Using $k = 0.1$

(a) use two iterations of the approximation formula $\left(\frac{dy}{dx}\right)_0 = \frac{y_1 - y_0}{h}$ to estimate the temperature of the coffee 3 minutes after it was put in the room.

(6)

The coffee in a different cup, which also had an initial temperature of 80 °C when it was put in the room, cools more slowly.

(b) Use this information to suggest how the value of k would need to be changed in the model.

(1)

a) $\theta_1 = \theta_0 + h\left(\frac{d\theta}{dt}\right)_0$ $t=0,\ \theta = 80$

$h = 1.5$

$\left(\frac{d\theta}{dt}\right)_0 = -0.1(80-20) = -6$

$\therefore \theta_1 = 80 + 1.5(-6) = 71°$

$\left(\frac{d\theta}{dt}\right)_1 = -0.1(71-20) = -5.1$

$\therefore \theta_2 = 71 + 1.5(-5.1) = \boxed{63.35°}$

b) A greater value of k corresponds to faster cooling.

$\therefore$ k should decrease, (but still be greater than 0.)

Question 2 continued

(Total for Question 2 is 7 marks)

P 6 0 1 5 2 A 0 5 1 6

3. Use algebra to find the values of x for which

$$\frac{x}{x^2 - 2x - 3} \leqslant \frac{1}{x+3}$$

(7)

$$\frac{x}{(x-3)(x+1)} \leqslant \frac{1}{x+3}$$

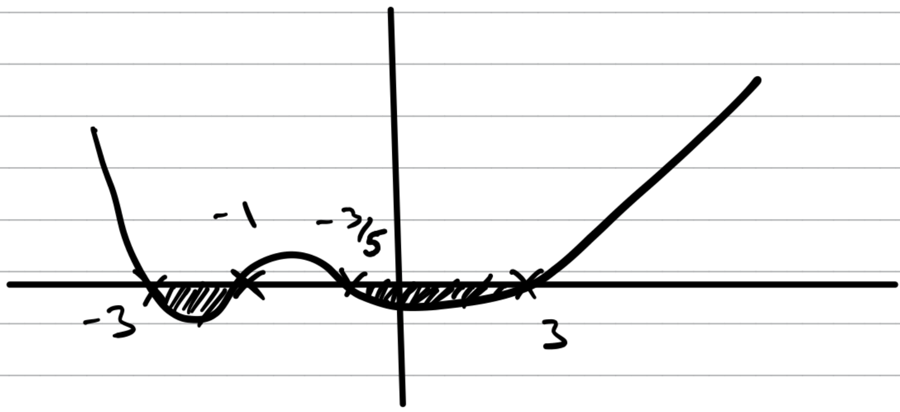
$$\frac{x(x+3)^2}{(x-3)(x+1)} \leqslant 1(x+3)$$

$$x(x+3)^2(x-3)(x+1) \leqslant (x+3)(x-3)^2(x+1)^2$$

$$(x+3)(x-3)(x+1)\left[x(x+3) - (x-3)(x+1)\right] \leqslant 0$$

$$(x+3)(x-3)(x+1)(x^2+3x-(x^2-2x-3)) \leqslant 0$$

$$(x+3)(x-3)(x+1)(5x+3) \leqslant 0$$



$\Rightarrow$ notice that for the original expressions given, $x=3$, $x=-1$, $x=-3$ are all undefined. can't be equal to

so required region is:

$$S = \{x \in \mathbb{R} \mid (-3 < x < -1) \cup (-\tfrac{3}{5} \leqslant x < 3)\}$$

Question 3 continued

(Total for Question 3 is 7 marks)

4. A scientist is investigating the properties of a crystal. The crystal is modelled as a tetrahedron whose vertices are $A(12, 4, -1)$, $B(10, 15, -3)$, $C(5, 8, 5)$ and $D(2, 2, -6)$, where the length of unit is the millimetre. The mass of the crystal is 0.5 grams.

(a) Show that, to one decimal place, the area of the triangular face ABC is $52.2\,\text{mm}^2$

(3)

(b) Find the density of the crystal, giving your answer in g cm^{-3}

(6)

a) $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 11 \\ -2 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -7 \\ 4 \\ 6 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 74 \\ 26 \\ 69 \end{pmatrix} = \begin{vmatrix} i & j & k \\ -2 & 11 & -2 \\ -7 & 4 & 6 \end{vmatrix}$

(cross product)

Area $\Delta ABC = \frac{1}{2}\left|\begin{pmatrix} 74 \\ 26 \\ 69 \end{pmatrix}\right|$

$= \frac{1}{2}\sqrt{74^2 + 26^2 + 69^2} = \boxed{52.2\,\text{mm}^2}$

b) $\overrightarrow{AD} = \begin{pmatrix} -10 \\ -2 \\ -5 \end{pmatrix}$

$\therefore$ Volume $= \frac{1}{6}\left|(\overrightarrow{AD}) \cdot (\overrightarrow{AB} \times \overrightarrow{AC})\right|$

$= \frac{1}{6}\left|\begin{pmatrix} -10 \\ -2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 74 \\ 26 \\ 69 \end{pmatrix}\right| = \frac{379}{2}\,\text{mm}^3$ //

tip: keep track of units

$= \frac{379}{2000}\,\text{cm}^3$ //

Density $= \frac{\text{mass}}{\text{volume}} = \frac{0.5}{\frac{379}{2000}} = \boxed{2.6\,\text{g/cm}^3}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

P 6 0 1 5 2 A 0 9 1 6

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. The rectangular hyperbola H has equation $xy = c^2$, where c is a non-zero constant.

The point $P\left(cp, \frac{c}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that an equation of the normal to H at P is

$$p^3x - py + c(1 - p^4) = 0$$

(4)

The normal to H at the point P meets H again at the point Q.

(b) Find the coordinates of the midpoint of PQ in terms of c and p, simplifying your answer where possible.

(6)

a) $y = c^2x^{-1}$ $\therefore \frac{dy}{dx} = -c^2x^{-2}$

$= \frac{-c^2}{(cp)^2} = -\frac{1}{p^2}$ //

$\therefore$ at normal, $m = p^2$ //

sub in $\left(cp, \frac{c}{p}\right)$ to $y - y_1 = m(x - x_1)$

$\Rightarrow y - \frac{c}{p} = p^2(x - cp)$

$\Rightarrow py - c = p^3(x - cp)$

$\Rightarrow py = p^3x + c - cp^4$

$\Rightarrow p^3x - py + c(1 - p^4) = 0$ //

b) $xy = c^2 \rightarrow y = \frac{c^2}{x}$

$\Rightarrow p^3x - p\left(\frac{c^2}{x}\right) + c(1 - p^4) = 0$

$\times x$

$\Rightarrow p^3x^2 - pc^2 + cx(1 - p^4) = 0$

$p^3x^2 + c(1 - p^4)x - pc^2 = 0$

Question 5 continued

$a = p^3, \quad b = c(1-p^4), \quad c = -pc^2$

Quadratic Formula: $x = \dfrac{-c(1-p^4) \pm \sqrt{c^2(1-p^4)^2 + 4p^4c^2}}{2p^3}$

$x = \dfrac{-c(1-p^4) \pm c\sqrt{1-p^8-2p^4+4p^4}}{2p^3}$

$x = c\left[\dfrac{p^4-1 \pm \sqrt{(p^4+1)^2}}{2p^3}\right]$

$x = c\left(\dfrac{p^4-1 \pm (p^4+1)}{2p^3}\right)$ //

$x_1 = c\left[\dfrac{2p^4}{2p^3}\right] = cp$ (this is P)

$x_2 = c\left[\dfrac{p^4-1-p^4-1}{2p^3}\right] = c\left[\dfrac{-4}{4p^3}\right]$

$= \dfrac{-c}{p^3}$ // (this is Q).

and $y = \dfrac{c^2}{x} = \dfrac{c^2}{-\frac{c}{p^3}} = -cp^3$ //

$\therefore Q\left(\dfrac{-c}{p^3}, -cp^3\right)$

//

Question 5 continued

midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So midpoint of PQ =

$$\boxed{\left(\frac{cp - \frac{c}{p^3}}{2}, \frac{\frac{c}{p} - cp^3}{2}\right)}$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS